Life of Fred

Calculus

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What Calculus Is About

I remember standing in the college bookstore at the beginning of my freshman year. I pulled a beginning calculus textbook off the shelf and opened it. What a frightening sight it was.

The pages were filled with strange symbolism like $\int_{x=0}^{2} (4 - x^2)\, dx$ and $\frac{\partial \phi}{\partial x} = M$. It might as well have been in Turkish. No one else in my family had ever studied calculus, so there was no one to give me an overview of what lay ahead. All I was told was that anyone who wanted to study any of the topics that I was even remotely interested in would want to have a grounding in calculus. Even business majors going on for a master's degree were required to study it.

But that didn't tell me what it was. I looked at some of the problems in that old textbook:

11. Find dy/dx for y = sin x.
18. Determine the eccentricity of $(y + 5)^2 + 4(x - 5)^2 = 1$.
22. Solve y' = tanh x

From my trig class I recognized "sin x" and knew that it didn't have theological overtones in this context. But I was still at a loss as to what calculus was about or why I needed to learn it. Was this stuff useful? Would I find a need for it in my everyday life?

Yes. The book you now hold in your hands shows that every aspect of calculus can arise in the course of daily living. If you've ever fallen into a vat of cheese soup (Chapter 19) or tried to run a thousand pounds of ammo through a custom's station (Chapter 9) you know what I mean.

So what's calculus? In a sentence:

If it moves at a varying speed,
if it has a curvy shape,
if it has a maximum that you'd like to find,
if it involves adding up an infinite number of terms,
then you're probably looking at calculus.

Contents

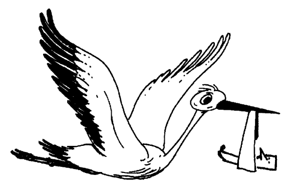
(Topics in *small italics* are covered in the *Further Ado* section of the book at the page numbers indicated.)

Chapter One

Functions

Once upon a time, a long time ago, on the western slopes of the Siberian mountains there lived Fred's parents. However, they weren't called Fred's parents since Fred hadn't been born yet.

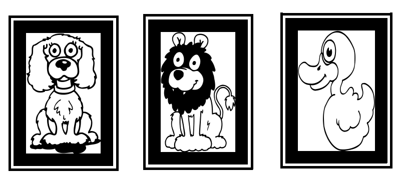
But one day, more recently than a long time ago, the stork delivered Fred. The lucky couple, Mr. & Mrs. Gauss, discovered that they were his parents.

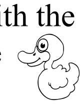


At least Mrs. Gauss (rhymes with "house") thought she was a parent. Staring at Fred, she chattered, "Oh, isn't our baby beautiful!"

Mr. Gauss frowned and said, "He doesn't look a bit like me." Mrs. Gauss didn't get the drift of what her husband was saying. She responded, "Of course he doesn't. He's just a little baby, all red and wrinkly, and, besides, he was very young when he was born." Fred's father rolled back his eyes, turned and left the room.

Mrs. Gauss carried him around a while and then, not knowing what to do with him, put him back in his crib. She had high hopes that her little tyke would grow up to be a country western singer. After she tucked him in, she handed him a new toy. It was a box with three buttons on it. Each button had an animal printed on it.



When Fred hit the button with the dog on it, the box sounded, "Bow-wow!" When he tried the lion, "Roar!" The duck, "Quack!" This was Fred's first encounter with the idea of function. He found out that EVERY time he touched the  he heard, "Quack!"

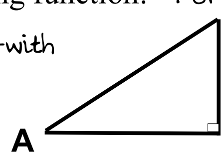
Here is how he summarized in his head what he knew about his animal-toy function:

> 1. There are two sets involved: the set of animal buttons {dog, lion, duck} and the set of animal sounds {Bow-wow, Roar, Quack}.
>
> 2. Every time I hit the lion I get a sound and it's always the same sound.

Fred was fascinated by this idea of function. You start with two sets and for each element of the first set there is exactly one element of the second set which corresponded to it. Fred looked around his study (crib) and invented a new function. His first set contained the things in his crib and the second set was colors. He saw his sheet and that matched up with "white" in the second set. His  matched up with "yellow." The bars on his crib also matched up with "yellow."

Can two different things be "yellow"? Yes. The only critical thing for the idea of a function is that each element in the first set have exactly one **image** in the second set. It's okay if two different elements in the first set have the same image.

Fred thought to himself, "This is baby stuff! I'm three days old and I should be able to think of a more sophisticated example of a function." He thought of his diaper which was in the shape of a triangle. He labeled one of the acute angles with the letter A and created the following function: "For any acute angle A, draw a diaper—I mean a triangle—with one of the acute angles being A.



Then," Fred continued, "measure the length of the side opposite and divide that length by the length of the hypotenuse." When Fred set angle A equal to 35º, the result of using his function (namely, drawing a triangle with a 35º angle and dividing the opposite side by the hypotenuse) gave him a result of 0.5735764. Fred was very good at measuring lengths. He called this function that he invented the sine function and he wrote $\sin(35^\circ) = 0.5735764$.

But what if he had used a bigger triangle? Would the answer come out differently? No. He knew he'd get the same answer every time since any two right triangles with 35º angles would be similar (something he had read in his geometry book on the previous day) and similar triangles are triangles in which the sides are proportional.

Now since every element in the first set, which is the set of all acute angles, has a unique image in the second set, Fred knew that he was dealing with a genuine function.

When Mrs. Gauss came in to see how Fred was doing, she found that he had drawn triangles all over his bed sheets.

As she looked down into Fred's study and made little "goo-goo-goo" sounds at him, he said, "Mom, let's play a little game. We'll call it, Guess the Function."

Fred continued, "I'm thinking of a function which I'm going to call 'f' and I'm going to give you some examples and you try and guess what the function is. Are you ready?"

Mrs. Gauss nodded but wasn't sure what Fred was talking about. Then Fred wrote on a sheet:

$f(7) = 15$

$f(3) = 7$

$f(6) = 13$

$f(100) = 201$

Mrs. Gauss looked at what he had written. She looked at it for a long time. Finally she said, "Are you hungry?"

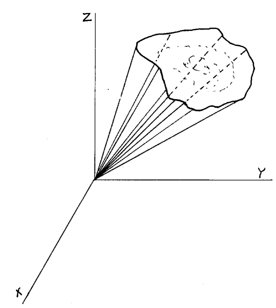
(From page 447)

Spherical Coordinates

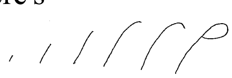
In rectangular coordinates $\int\int\int$ dV was $\int\int\int$ dx dy dz. Great for everyday use. A perennial favorite.

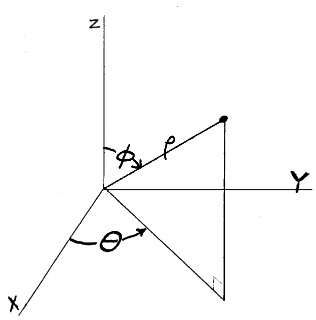
In cylindrical coordinates, it was $\int\int\int$ r dr dθ dz. That's a real handy way to approach a piece of pie even when the outer edge is wavy (when r is a function of θ).

But there are times when spherical coordinates is the way to go. The tip-off is when you have some surface and all the points between that surface and the origin. It's like an ice cream cone where the sides of the cone may be fluted.



To locate a point in space in spherical coordinates, first measure the distance from the origin to the point. Call that distance ρ (rho). That's the Greek letter for *r* . I know it looks like the Latin letter p but you and I know that p in Greek is π. To write a ρ, start at the bottom. Here's the movie:





(Some books call that distance *r* but that's asking for trouble, since we use *r* in cylindrical coordinates for a different distance.)

Next, the angle that ρ makes with the z axis is called ϕ (phi). Finally, θ keeps the same meaning it had with cylindrical coordinates (viz., the angle between the x axis and the line from the origin to the projection of the point on the x-y plane).

So we have the point (ρ, ϕ, θ).

Index